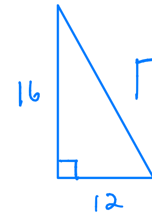
Mini-Lecture 9.1

The Pythagorean Theorem and Its Converse

Learning Objectives:

1. Use the Pythagorean Theorem.
2. Use the Converse of the Pythagorean Theorem.
3. Key vocabulary: *Pythagorean triple*

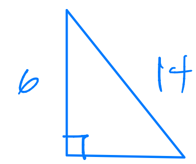


$\sqrt{16^2+12^2} = \sqrt{256+144}$
$= \sqrt{400}$
$= 20$

Key Examples:

1. a) The legs of a right triangle have lengths 12 and 16. Find the length of the hypotenuse.
   b) Check to see that the side lengths in part a) form a Pythagorean triple.

2. The hypotenuse of a right triangle has length 14. One leg has length 6. Find the length of the other leg. Write the answer in simplest radical form.



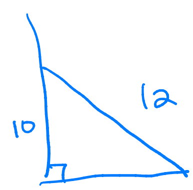
$\sqrt{196-36}$
$= \sqrt{160}$
2 | 80
2 | 40
2 | 20
2 | 10
$= 4\sqrt{10}$

3. A painter leans a 12-foot ladder against a wall. The ladder reaches 10 feet up the wall. To the nearest tenth of a foot, how far is the base of the ladder from the wall?

4. A triangle has side lengths 25, 45, and 50. Is the triangle a right triangle? Explain.

5. Is a triangle with side lengths 11, 14, and 18 acute, obtuse, or right?

3



$\sqrt{12^2-10^2} = \sqrt{44} = 2\sqrt{11} \approx 6.6332$

4 50 is largest
Does $50^2 = 25^2 + 45^2$?
$2500 < 2650$ so acute

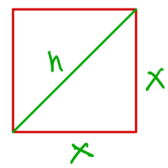
(By law of Cosines, $\cos^{-1}\left(\frac{25^2+45^2-50^2}{2(25)(45)}\right) \approx 86.177°$)

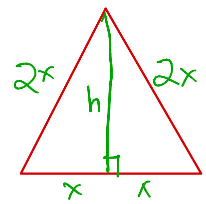
5 18 is largest; Does $18^2 = 11^2 + 14^2$?
$324 > 317$ So obtuse

Answers: 1a) 20 1b) $12^2 + 16^2 = 144 + 256 = 400 = 20^2$ 2) $4\sqrt{10}$ 3) 6.6 ft
4) No; $25^2 + 45^2 = 625 + 2025 = 2650$, and $50^2 = 2500$, so $25^2 + 45^2 \neq 50^2$. 5) obtuse

Mini-Lesson 9.2

Special Right Triangles





Learning Objectives:

1. Use the properties of 45°-45°-90° triangles.
2. Use the properties of 30°-60°-90° triangles.

$h^2 = x^2 + x^2$
$h^2 = 2x^2$
$h = x\sqrt{2}$

$h^2 + x^2 = (2x)^2$
$h^2 = 4x^2 - x^2 = 3x^2$
$h = x\sqrt{3}$

Key Examples:

1. Find the length of the hypotenuse of a 45°-45°-90° triangle with leg length $13\sqrt{2}$.

   $(13\sqrt{2})(\sqrt{2}) = (13)(2) = 26$

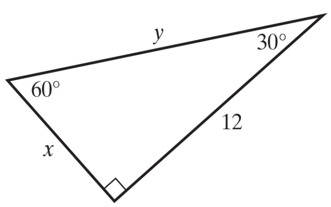
2. The length of the hypotenuse of a 45°-45°-90° triangle is 22. Find the length of one leg.

   $x\sqrt{2} = 22, \quad x = \frac{22 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{22\sqrt{2}}{2} = 11\sqrt{2}$

3. A playground sits on a square lot with a perimeter of 600 feet. Two diagonal paths cross the lot. To the nearest foot, how long is each path?

   $4x = 600$
   $x = 150$
   $x = 150\sqrt{2}$

4. Find the values of x and y. If an answer involves a radical, write it in simplest radical form.

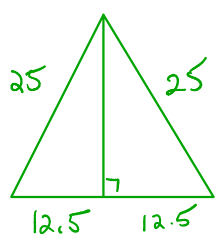


$x\sqrt{3} = 12$

$x = \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$

$y = 2x = 8\sqrt{3}$

5. A road sign has the shape of an equilateral triangle. If the sides of the sign measure 25 in., what is the height of the sign to the nearest tenth of an inch?



$x = 12.5$
$x\sqrt{3} = 12.5\sqrt{3} = \frac{25\sqrt{3}}{2}$

Answers: 1) 26 2) $11\sqrt{2}$ 3) 212 ft 4) $x = 4\sqrt{3},\ y = 8\sqrt{3}$ 5) 21.7 in.

Mini-Lesson 9.3

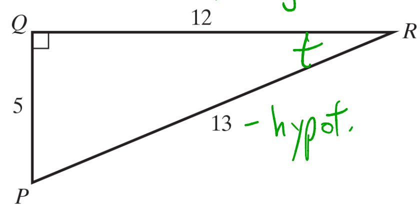
Trigonometric Ratios

Learning Objectives:

1. Use the sine, cosine, and tangent ratios to determine side lengths in right triangles.
2. Use the sine, cosine, and tangent ratios to determine angle measures in right triangles.
3. Key vocabulary: *trigonometric ratios, sine, cosine, tangent*

Key Examples:

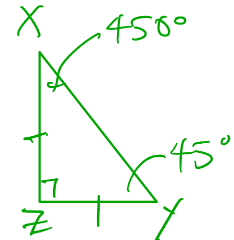
1. What are the sine, cosine, and tangent ratios for $\angle R$? Give the exact value and a four-decimal place approximation.



$\sin R = \frac{5}{13}$

$\cos R = \frac{12}{13}$

$\tan R = \frac{5}{12}$



2. In right triangle XYZ, $\angle Z$ is the right angle and $\sin X = \sin Y$. What are $m\angle X$ and $m\angle Y$?

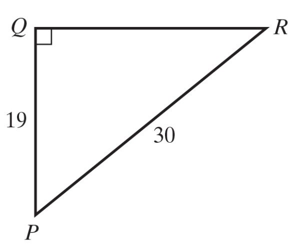
3. An airplane rises from takeoff and flies at an angle of 12° with the horizontal runway. When it has flown 3500 feet, how far, to the nearest foot, is it above the ground?

$\frac{h}{3500} = \sin 12°$

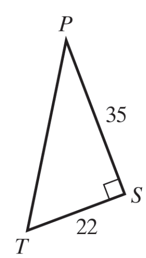
$h = 3500 \sin 12°$

4. What is $m\angle P$ to the nearest degree?

a)



b)



$\frac{22}{35} = \tan P$

$P = \tan^{-1}\left(\frac{22}{35}\right)$

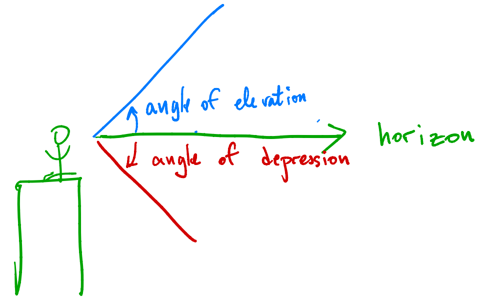
$\frac{19}{30} = \cos P$

$P = \cos^{-1}\left(\frac{19}{30}\right)$

Answers: 1) $\sin R = \frac{5}{13} \approx 0.3846$, $\cos R = \frac{12}{13} \approx 0.9231$, $\tan R = \frac{5}{12} \approx 0.4167$ 2) $m\angle X = m\angle Y = 45°$ 3) 728 ft

4a) 51° 4b) 32°

Mini-Lesson 9.4
Solving Right Triangles



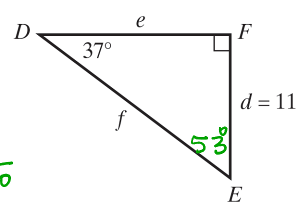
Learning Objectives:

1. Solve right triangles.
2. Use angles of elevation and depression to solve problems.
3. Key vocabulary: *solving the triangle, angle of elevation, angle of depression*

Key Examples:

1. Solve the right triangle. If needed, round any answers to one decimal place.

a)



$E = 90° - 37°$

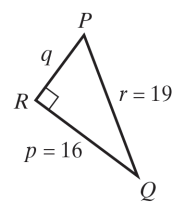
$\frac{e}{11} = \tan 53°$

$e = 11 \tan 53°$

$\frac{11}{f} = \sin 37°$

$f = \frac{11}{\sin 37°}$

b)



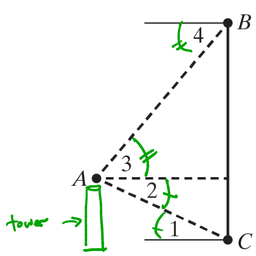
$q^2 + 16^2 = 19^2$

$q = \sqrt{19^2 - 16^2} = \sqrt{105}$

$\frac{16}{19} = \sin P,\quad P = \sin^{-1}\left(\frac{16}{19}\right)$

$\frac{16}{19} = \cos Q,\quad Q = \cos^{-1}\left(\frac{16}{19}\right)$

2. An observer standing on a platform at point A is making measurements to the top of a tower at point B and to the base of the tower at point C.



What is a description of the angle as it relates to this situation?

a) $\angle 1 \cong \angle 2$ (A.I.) angle of depression
b) $\angle 2$ = angle of depression
c) $\angle 3$ angle of elevation
d) $\angle 4 \cong \angle 3$ (AI) angle of elevation

3. Jason is standing 24 ft from the base of a tree. From his eye level, 6 ft above the ground, the angle of elevation of the top of the tree is 64°. Find the height of the tree to the nearest foot.

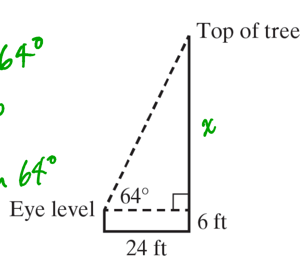
$\frac{x}{24} = \tan 64°$

$x = 24 \tan 64°$

height is

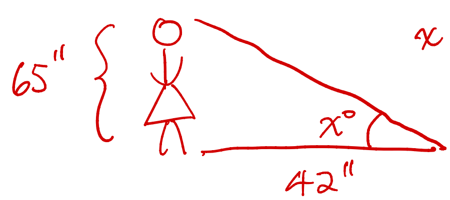
$6 + 24 \tan 64°$

feet tall



$\tan x° = \frac{65}{42}$

$x = \tan^{-1}\left(\frac{65}{42}\right)$



4. Meghan, who is 65 inches tall, casts a shadow 42 inches long. Find the angle of elevation of the sun to the nearest degree.

Answers: 1a) $m\angle E = 53°$, $e \approx 14.6$, $f \approx 18.3$ 1b) $q = 10.2$, $m\angle P = 57.4°$, $m\angle Q = 32.6°$ 2a) angle of elevation from base of tower to observer 2b) angle of depression from observer to base of tower 2c) angle of elevation from observer to top of tower 2d) angle of depression from top of tower to observer 3) 55 ft 4) 57°

Mini-Lesson 9.5
Vectors

Learning Objectives:

1. Describe vectors. have magnitude & direction
2. Solve problems involving vector addition.
3. Key Vocabulary: *vector, magnitude, initial point, resultant, component form*

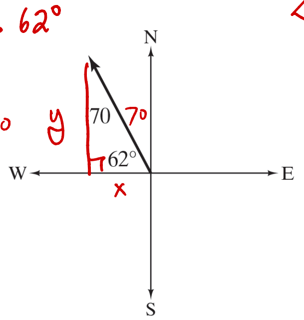
Key Examples:

1. What is the component form of the vector in the figure? Round the coordinates to the nearest tenth.

$\langle$ horizontal , vertical $\rangle$

$\frac{x}{70} = \cos 62°$

$\frac{y}{70} = \sin 62°$



$\langle 70 \cos 62°, 70 \sin 62° \rangle$

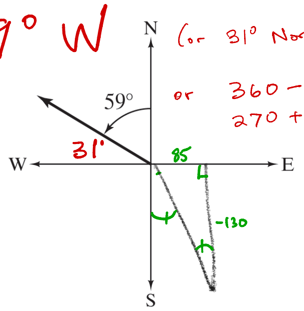
$70 \langle \cos 62, \sin 62 \rangle$

2. a) Describe the direction of the vector in the figure using compass directions.
   b) Is there more than one way to describe the direction of this vector? Explain.

N 59° W

(or 31° North of West

or $360 - 59 = 301°$
$270 + 31 = 301°$



$\langle 85, -130 \rangle$

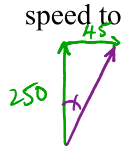
magnitude is $\sqrt{85^2 + 130^2}$

direction: S $\tan^{-1}\left(\frac{85}{130}\right)$ E

3. An airplane lands 85 km east and 130 km south from where it took off. What are the approximate magnitude and direction of the flight vector?

4. What is the resultant of $\langle -6, 8 \rangle$ and $\langle -2, -3 \rangle$? $= \langle -6-2, 8-3 \rangle = \langle -8, 5 \rangle$

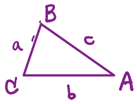
5. A small airplane has a speed of 250 (magnitude) mph in still air. The plane heads directly north and encounters a 45-mph wind blowing due east. Find the resulting speed and direction of the plane. Round the speed to the nearest whole number and the angle to the nearest degree.



speed : $\sqrt{250^2 + 45^2}$

direction: N $\tan^{-1}\left(\frac{45}{250}\right)$ E

Answers: 1) $\langle -32.9, 61.8 \rangle$ 2a) 59° west of north 2b) Yes; it can also be described as 31° north of west.
3) about 155 km at 57° south of east 4) $\langle -8, 5 \rangle$ 5) 254 mph at 10° east of north

for ANY Δ: 

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Extension Mini-Lesson

The Law of Sines

Learning Objectives:

1. Use the Law of Sines to solve oblique triangles.
2. Use the Law of Sines to solve, if possible, the triangle or triangles in the $\boxed{\text{ambiguous case}}$. SSA
3. Find the area of an oblique triangles using the sine function.
4. Key Vocabulary: *oblique triangle, Law of Sines, ambiguous case*

no right angle: all acute or 2 acute one obtuse angle

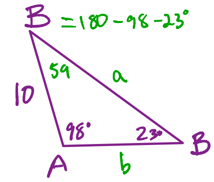
Key Examples:

1. Solve triangle *ABC* with $A = 98°$, $B = 23°$, and $b = 10$ inches. Round lengths of sides to the nearest tenth. AAS
2. Solve triangle *ABC* with $B = 72°$, $C = 36.3°$. and $a = 18$. Round measures to the nearest tenth. ASA

SSA "ambiguous case"

3. 1 solution — Solve triangle *ABC* with $A = 108°$, $a = 41$, and $b = 27$. Round lengths of sides to the nearest tenth and angle measures to the nearest degree. SSA: No solution / 1 solution / 2 solution
4. 0 solutions — Solve triangle *ABC* with $B = 68°$, $b = 15$, and $c = 18$. SSA
5. 2 solutions — Solve triangle *ABC* with $C = 47°$, $b = 31$, and $c = 26$. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.
6. Find the area of a triangle having two sides of lengths 32 centimeters and 45 centimeters and an included angle of 118°. Round to the nearest square centimeter.

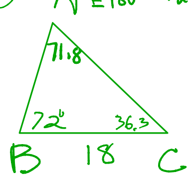
① $B = 180 - 98 - 23°$



$\frac{a}{\sin 98} = \frac{10}{\sin 23}$, $a = \frac{10 \sin 98°}{\sin 23}$

$b = \frac{10 \sin 59}{\sin 23}$

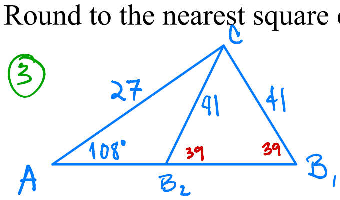
② $A = 180 - 72 - 36.3$



$\frac{b}{\sin 72} = \frac{18}{\sin 71.8}$, $b = \frac{18 \sin 72}{\sin 71.8}$

$c = \frac{18 \sin 36.3}{\sin 71.8}$

③



$\frac{\sin 108°}{41} = \frac{B_1}{27}$, $B_1 = \sin^{-1}\left(\frac{27 \sin 108°}{41}\right)$

$= 38.778$

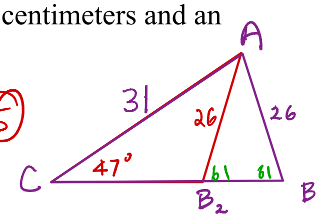
$C_1 = 180 - A - B$

$B_2 = 180 - 39 = 141$

but $108 + 141 > 180$

one solution only

⑤



$\frac{\sin 47}{26} = \frac{B_1}{31}$,

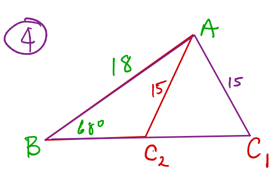
$B_1 = \sin^{-1}\left(\frac{31 \sin 47}{26}\right) = 60.6917...$ B

$C_1 = 180 - 47 - B_1 = 72.3082...$ C

$\frac{c}{72.308...} = \frac{26}{\sin 74}$, $c = 33.869164...$

$B_2 = 180 - 61 = 119°$

④



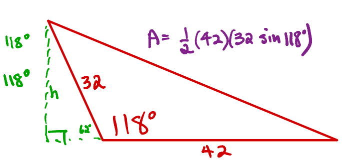
$\frac{\sin 68}{15} = \frac{\sin C}{18}$

$\frac{18 \sin 68}{15} > 1$

⑥

$\frac{h}{32} = \sin 62° = \sin 118°$

$h = 32 \sin 118°$



$A = \frac{1}{2}(42)(32 \sin 118°)$

Answers: 1) $C = 59°$, $a \approx 25.3$ inches, $c \approx 21.9$ inches 2) $A = 71.7°$, $b \approx 18.0$, $c \approx 11.2$ 3) $B \approx 39°$, $C \approx 33°$, $c \approx 23.6$ 4) no triangle 5) $A_1 \approx 72°$, $B_1 = 61°$, $a_1 \approx 33.9$ and $A_2 \approx 14°$, $B_2 = 119°$, $a_2 \approx 8.4$ 6) 636 sq cm

Extension Mini-Lesson

The Law of Cosines

for SAS, SSS

Learning Objectives:

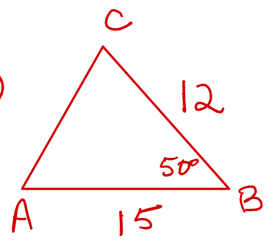
1. Use the Law of Cosines.
2. Key Vocabulary: *Law of Cosines*

Key Examples:

1. Solve triangle ABC with $B = 50°$, $a = 12$, and $c = 15$. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

2. Solve triangle ABC with $a = 13$, $b = 25$, and $c = 33$. Round angle measures to the nearest degree.

①



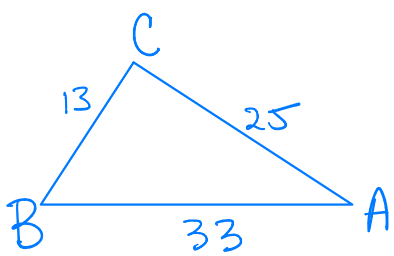
$$b = \sqrt{15^2 + 12^2 - 2(15)(12)\cos 50°} = \sqrt{137.5694605} \approx 11.73013177$$

STO → $\boxed{\text{ALPHA}}$ B

$$A = \cos^{-1}\left(\frac{15^2 + B^2 - 12^2}{2(15)(B)}\right) = 51.5975633$$

$$C = 180 - 50 - A$$

②



$$A = \cos^{-1}\left(\frac{33^2 + 25^2 - 13^2}{2(33)(25)}\right)$$

$$B = \cos^{-1}\left(\frac{13^2 + 33^2 - 25^2}{2(13)(33)}\right)$$

$$C = \cos^{-1}\left(\frac{13^2 + 25^2 - 33^2}{2(13)(25)}\right)$$

Answers: 1) $b \approx 11.7$, $A \approx 52°$, $C \approx 78°$ 2) $C \approx 117°$, $A \approx 21°$, $B \approx 42°$